Closing Tue: 1.6, 2.1
Closing Thur: 2.2 (big assignment!)
Midterm 1 will be returned Tuesday.

skip?

## 2.1 Quadratic Functions Intro

*Entry Task: See how many of these you can do in 5 minutes…*

1. Expand out

(a) $(x+5)(x-6) = x^2+5x-6x-30$
$= x^2 - x - 30$
$(5)+(-6)$  $(5)\cdot(-6)$

(b) $(x-7)^2 = (x-7)(x-7) = x^2-7x-7x+49$
$= x^2-14x+49$

(c) $(A+B)^2 = A^2+2AB+B^2$

2. Factor (fill in the numbers):

$x^2 - 8x + 16 = (x-4)(x-4)$
$(-4)+(-4)$  $(-4)(-4)$

$x^2+9x+20 = (x+4)(x+5)$
$4+5$  $(4)\cdot(5)$

3. Solve

(a) $3q^2 - 1 = 11$

(b) $(z-1)^2 = 7$

(a) $3q^2 = 12$
$q^2 = 4$
$\boxed{q = \pm 2}$

CHECK! $3(2)^2-1 = 11$ ✓
$3(-2)^2-1 = 11$ ✓

(b) $z - 1 = \pm\sqrt{7}$
$\boxed{z = 1-\sqrt{7} \text{ or } z = 1+\sqrt{7}}$

CHECK: $(1-\sqrt{7}-1)^2 = 7$ ✓
$(1+\sqrt{7}-1)^2 = 7$ ✓

A **quadratic function** can be written in the form:

"LINEAR TERM" "CONSTANT TERM" "QUADRATIC TERM"

$$y = ax^2 + bx + c.$$

The graph of a quadratic function is called a *parabola*.

CALLED A COEFFICIENT

Examples:

$$y = -5x^2 + 20x + 30.$$

$a=-5$, $b=20$, $c=30$

$$f(x) = 4 + 2x^2.$$

$a=2$, $b=0$, $c=4$

$$P(q) = (10q - 5q^2) - (3q + 6).$$

$= 10q - 5q^2 - 3q - 6$

$= -5q^2 + 7q - 6$

$a=-5$, $b=7$, $c=-6$

*Two Motivating Examples*

1. **TR from demand**:

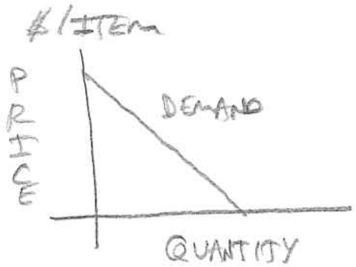
If $x$ = quantity, and

$p = 105 - 0.1x$ = price (demand)

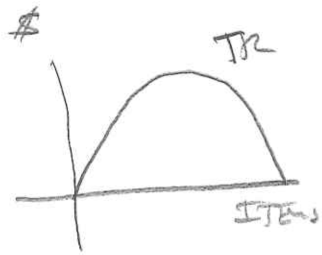
then

$TR(x) = (105 - 0.1x)x$ (why?)

$= 105x - 0.1x^2$ (why?)




$\Rightarrow$




Example)

IF WE SELL $x = 100$ ITEMS

THEN THE DEMAND PRICE MUST HAVE BEEN

$p = 105 - 0.1(100) = 105 - 10 = \$95$/ITEM

SO

TOTAL REVENUE $= \underbrace{\$95/\text{ITEM}}_{(105-0.1x)} \cdot \underbrace{100 \text{ ITEMS}}_{x} = \$9500$

SO PATTERN IS $TR(x) = (105 - 0.1x)x$

2. **Projectiles**:

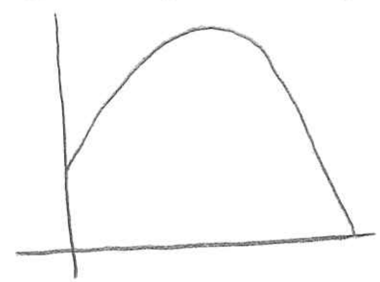
A ball is thrown in the air from an initial height of 6 ft with an initial upward velocity of 20 ft/s.

INITIAL HEIGHT INITIAL VELOCITY

The height in feet is given by

$$h(t) = \underset{c}{6} + \underset{b}{20t} - \underset{a}{\cancel{32}\,16\,t^2}$$

Comes from gravity

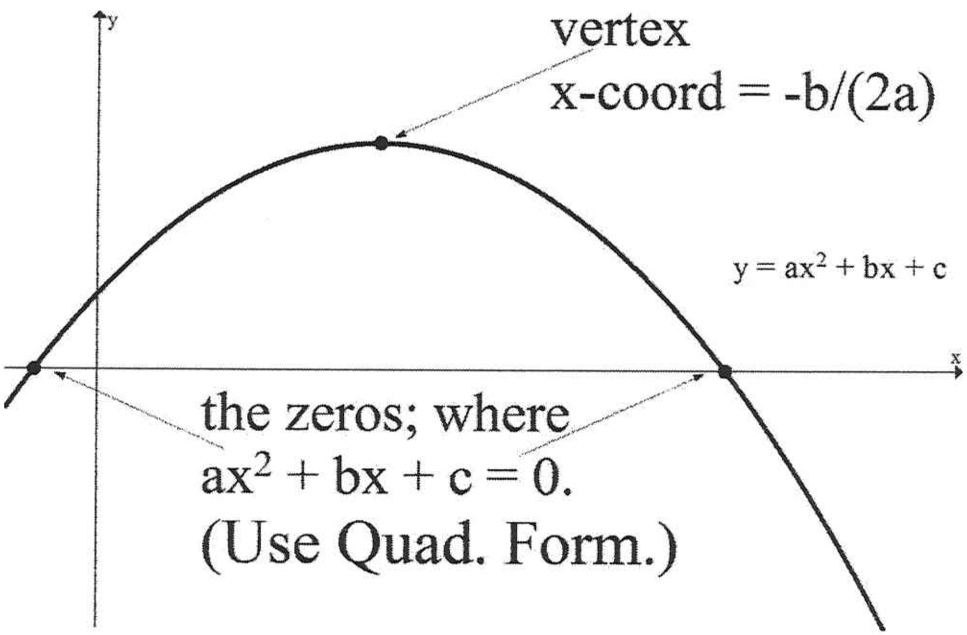


**Parabola Basics**

If *a* is negative, it opens downward.

a b c

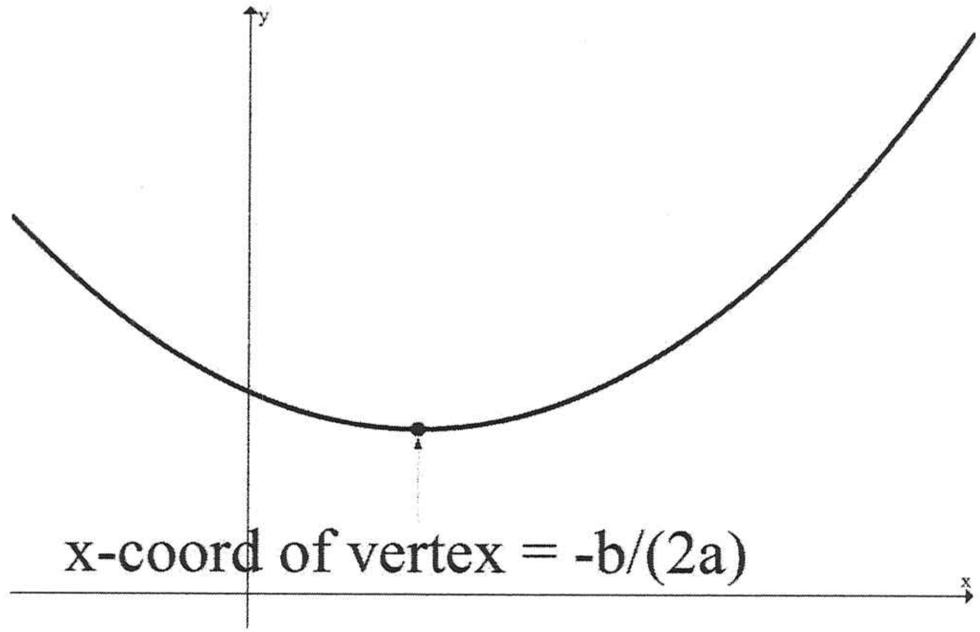
*Example*: $y = -5x^2 + 20x + 30.$




If *a* is positive, it opens upward.

a b c

*Example*: $y = 2x^2 + 28x + 4.$




The solution(s) to $ax^2 + bx + c = 0$ are given by the *quadratic formula*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The x-coordinate of the vertex is given by

$$x = -\frac{b}{2a}$$

**Solving quadratic equations**:

Given a *quadratic* ***equation…***

1. Simplify/Clear denominators.

2. Subtract to make one side zero.
   You will have something like:
$$ax^2 + bx + c = 0$$

3. Use the quadratic formula
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

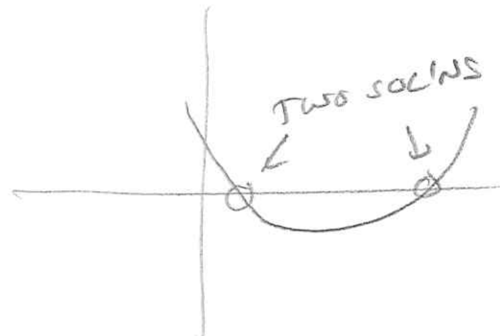
*Note (looking under the radical)*:

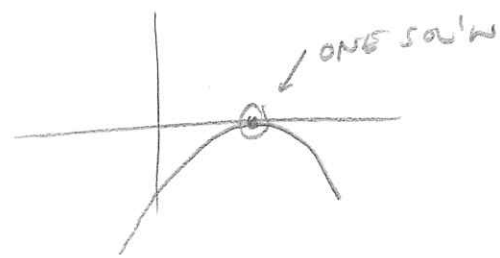
If $b^2 - 4ac > 0$, then two solutions.

If $b^2 - 4ac = 0$, then one solution.

If $b^2 - 4ac < 0$, then no solutions.

Note: When using this, the ***equation*** with have only ONE variable, an equal sign, and *NO y or f(x).* The other side will be zero!




Solve:

1. $x^2 - 7x = 0$

$\boxed{\text{OPTION 1}}$ FACTOR

$x(x-7)=0$

$\boxed{x=0}$ OR $\boxed{x=7}$

CHECK!

OPTION 2 $a=1, b=-7, c=0$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(0)}}{2(1)} = \frac{7 \pm \sqrt{49}}{2} = \frac{7 \pm 7}{2}$$

$\boxed{x = \frac{7-7}{2} = 0}$ OR $\boxed{x = \frac{7+7}{2} = 7}$

CHECK!

2. $7 + 2x - 2x^2 = 4 + x$ $\quad -4 \quad -x$

$-2x^2 + x + 3 = 0$

FACTOR?? SKIP TO OPTION 2

$\boxed{\text{OPTION 1}}$:

$(2x+3)(-x+1)=0$

$2x+3=0$ OR $-x+1=0$

$\boxed{\text{OPTION 2}}$ $a=-2, b=1, c=3$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(-2)(3)}}{2(2)} = \frac{-1 \pm \sqrt{1+24}}{4} = \frac{-1 \pm \sqrt{25}}{4}$$

$\boxed{x = \frac{-1+5}{4} = 1}$ OR $\boxed{x = \frac{-1-5}{4} = \frac{-6}{4} = -\frac{3}{2}}$

CHECK!

3. $\frac{x}{3} - 4x^2 = 2x - 1$ $\quad \cdot 3$

$x - 12x^2 = 6x - 3$ $\quad +12x^2 \quad -x$

$0 = \underset{a}{12}x^2 + \underset{b}{5}x \underset{c}{-3}$

$\boxed{\text{OPTION 2}}$ $a=12, b=5, c=-3$

$$x = \frac{-(5) \pm \sqrt{(5)^2 - 4(12)(-3)}}{2(12)} = \frac{-5 \pm \sqrt{25+144}}{24}$$

$$= \frac{-5 \pm \sqrt{169}}{24} = \frac{-5 \pm 13}{24}$$

$$\boxed{x = \frac{-5-13}{24} = -\frac{18}{24} = -\frac{3}{4} \quad \text{OR} \quad x = \frac{-5+13}{24} = \frac{9}{24} = \frac{3}{8}}$$

CHECK!

**Three ways to solve quad. equations.**

## 1. *Factoring*

$$2x^2 + 12x + 40 = 54$$
$$x^2 + 6x + 20 = 27$$
$$x^2 + 6x - 7 = 0$$
$$(x-1)(x+7) = 0$$

Thus, $x - 1 = 0$ or $x + 7 = 0$

$$x = 1 \quad \text{or} \quad x = -7$$

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## 2. *Completing the Square*

$$2x^2 + 12x + 40 = 54$$
$$x^2 + 6x + 20 = 27$$

Completing the square gives

$$x^2 + 6x + 9 - 9 = 7$$
$$(x+3)^2 - 9 = 7$$
$$(x+3)^2 = 16$$
$$x + 3 = 4 \quad \text{or} \quad x + 3 = -4$$
$$x = 1 \quad \text{or} \quad x = -7$$

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## 3. *Quadratic Formula*:

$$2x^2 + 12x + 40 = 54$$
$$x^2 + 6x + 20 = 27$$
$$x^2 + 6x - 7 = 0$$
$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(-7)}}{(2(1))}$$
$$= \frac{-6 \pm \sqrt{26 + 28}}{2}$$
$$= \frac{-6 \pm \sqrt{64}}{2} = \frac{-6 \pm 8}{2}$$

Thus,

$$x = \frac{-6+8}{2} = 1 \text{ or } x = \frac{-6-8}{2} = -7$$

**You can use any of these! Just check your final answer.**

***Proving the Quadratic Formula***

(not required, for your own interest)

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Completing the square:

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} + \frac{c}{a} = 0$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Side note: Splitting up the fraction gives

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

So

$$x = -\frac{b}{2a}$$

is halfway between the two roots! (it is the x-coordinate of the vertex)

**Method for finding the vertex**:

Given a *quadratic* ***function*** such as

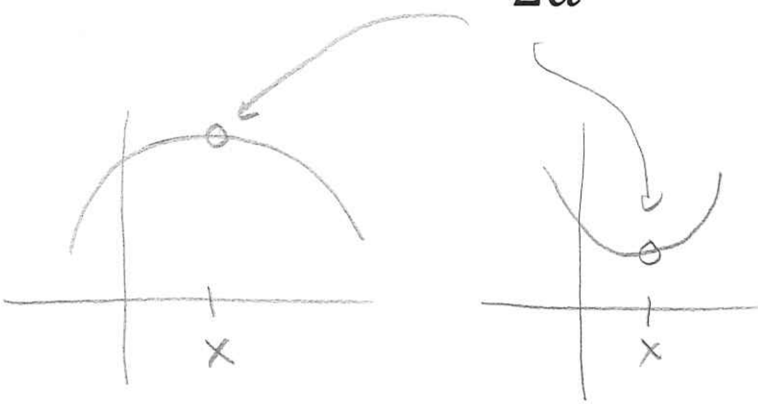
$$y = ax^2 + bx + c$$

or

$$f(x) = ax^2 + bx + c$$

The *x-coordinate of the vertex* is at:

$$x = -\frac{b}{2a}$$



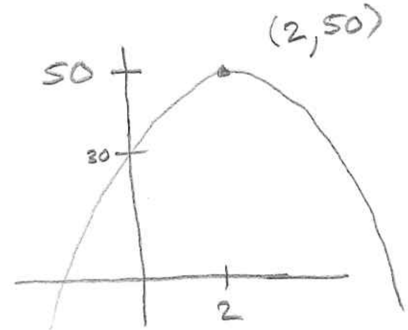

Note: When using this, we have a ***function***, not an equation. There is NOT a zero on one side. Instead there is a function name, *f(x)*, or a *y*. That is, there are TWO variables x and y present before we use the vertex formula!

Find the *x* and *y* coordinates of the vertex:

1. $y = \overset{c}{30} - \overset{a}{5}x^2 + \overset{b}{20}x.$ $\begin{cases} a = -5 \\ b = 20 \\ c = 30 \end{cases}$

$$x = -\frac{b}{(2a)} = -\frac{20}{(2(-5))} = -\frac{20}{-10} = 2$$

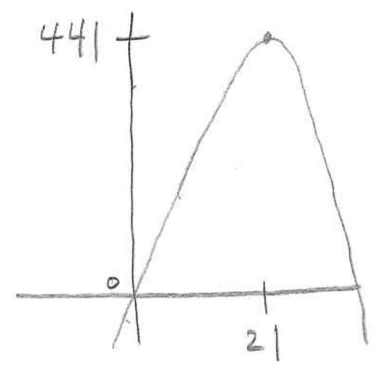
$$y = 30 - 5(2)^2 + 20(2)$$
$$= 30 - 20 + 40 = 50$$




2. $y = 42x - x^2.$ $\begin{cases} a = -1 \\ b = 42 \\ c = 0 \end{cases}$

$$x = -\frac{b}{(2a)} = -\frac{42}{(2(-1))} = \frac{-42}{-2} = 21$$

$$y = 42(21) - (21)^2 = 882 - 441 = 441$$

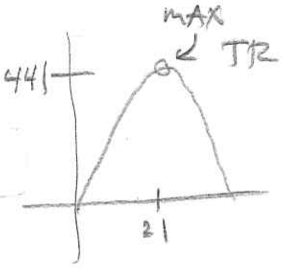


*Example*: (A preview of an application) Suppose total revenue (TR) and total cost (TC) are given by

$R(x) = 42x - x^2$ and $C(x) = 50 + 3x$

where x is in hundred items and $R(x)$, $C(x)$ are in hundred dollars.

(a) Revenue is a parabola that opens downward. So the maximum is at its vertex.




$a = -1, b = 42, c = 0$

$$\boxed{\begin{gathered} x = -\frac{b}{(2a)} = -\frac{(42)}{2(-1)} = 21 \\ R(21) = 42(21) - (21)^2 = 441 \end{gathered}}$$

(a) What quantity maximizes Revenue? And what is the max Revenue?

(b) Find the break-even points (*i.e.* quantities where profit is zero). *This is not the same as breakeven price!*

(c) What quantity maximizes profit?

(b) $R(x) \overset{?}{=} C(x)$

$42x - x^2 \overset{?}{=} 50 + 3x$

$0 = x^2 - 39x + 50$

$$x = \frac{-(-39) \pm \sqrt{(39)^2 - 4(1)(50)}}{2(1)}$$

$$= \frac{39 \pm \sqrt{1521 - 200}}{2} \approx \frac{39 \pm 36.34556}{2}$$

$$x = \frac{39 + 36.34556}{2} = \frac{75.34556}{2} \approx 37.67278$$

$\boxed{x = 37.67 \text{ HUNDRED ITEMS}}$

OR

$$x = \frac{39 - 36.34556}{2} \approx \frac{2.654436}{2} \approx 1.3272$$

$\boxed{x = 1.33 \text{ HUNDRED ITEMS}}$

(c) PROFIT = TR(x) − TC(x)

$= (42x - x^2) - (50 + 3x) = 42x - x^2 - 50 - 3x = -x^2 + 39x - 50$

MAX IS AT VERTEX $\quad x = \frac{-39}{2(-1)} = \boxed{19.5 \text{ HUNDRED ITEMS}}$